Math 270 HW#7

Name:_____ Date:_____

Show your work very clearly, neatly, and box your final answer.

One Side Only

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1. Prove that if A is idempotent, then A^T is idempotent.

2. Prove that if $\mathbf{A}$ is nonsingular, then so is $\mathbf{A}^{\mathbf{T}}$.

3. Prove if A is a symmetric nonsingular matrix is, then A^{-1} is also symmetric.

4. Suppose $\mathbf{A}$ is a square matrix satisfying the equation $\mathbf{A^3 - 2I = 0}$. Show that $\mathbf{A - I}$ is invertible and find the inverse. **Hint: Factoring of difference of two cubes.**

5. Suppose that $\mathbf{A}$ is an $n \times n$ matrix and $\mathbf{B}$ is an invertible $n \times n$ matrix, use mathematical induction to prove that $\left(\mathbf{BAB^{-1}}\right)^n = \mathbf{BA}^n\mathbf{B}^{-1}$ where n is a natural number.

6. If A is a nonsingular matrix and B is row-equivalent to A, then B is nonsingular.

7. Find the $LU-$factorization of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ -2 & 1 & -1 & 0 \\ 6 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

8. Use the $LU-$factorization to solve $\begin{cases} 2x & +y & +z & -t & =7 \\ & 3y & +z & -t & =-3 \\ & & -2z & & =2 \\ 2x & +y & +z & -2t & =8 \end{cases}$