

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Show your work very clearly, neatly, and box your final answer.**

**One Side Only**

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1. If  $C$  is an invertible matrix such that  $CA = CB$ , then  $A = B$ .

2. Prove that if  $A$  is idempotent, then  $A^T$  is idempotent.

3. Prove if  $A$  is an idempotent nonsingular matrix of order  $n$  then  $A = I_n$ .

4. Prove if  $A$  is a symmetric nonsingular matrix is, then  $A^{-1}$  is also symmetric.

5) What are Elementary Matrices, and what are they used for? Give an example.

6) Let  $A$ ,  $B$ , and  $A + B$  be nonsingular matrices. Show that

$$\left(A^{-1} + B^{-1}\right)^{-1} = A(A + B)^{-1} B.$$

$$7) \text{ Given: } \begin{cases} x_1 + x_2 + 2x_3 = 0 \\ x_1 - x_2 + x_3 = -1 \\ 2x_1 + x_2 + x_3 = 2 \end{cases}$$

a) Write in  $Ax = b$  form.

b) Find  $A^{-1}$

c) Use  $A^{-1}$  to solve for  $x$ .

**8)** Prove that if  $I - AB$  is nonsingular, then so is  $I - BA$ .

Hint: Multiply by  $I + BCA$  where  $C = (I - AB)^{-1}$ .

**9)** If  $A$  and  $B$  are  $n \times n$  matrices and  $A$  is invertible, then prove that  $(ABA^{-1})^2 = AB^2A^{-1}$ .