

Name: _____ Date: _____

Show your work very clearly, neatly, and box your final answer.**One Side Only**

1. Given $\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$, find

a) \mathbf{A}^T

b) \mathbf{A}^{-1}

c) $(\mathbf{A}^{-1})^T$

d) $(\mathbf{A}^T)^{-1}$

2. Show If $\mathbf{A}^2 - 2\mathbf{A} + 5\mathbf{I} = \mathbf{0}$, then $\mathbf{A}^{-1} = \frac{1}{5}(\mathbf{2I} - \mathbf{A})$.

3. Let \mathbf{A} and \mathbf{B} be two $n \times n$ symmetric matrices, Prove that if \mathbf{AB} is symmetric matrix, then $\mathbf{AB} = \mathbf{BA}$.

4. Prove if \mathbf{A} & \mathbf{B} are idempotent of order $n \times n$ and $\mathbf{AB} = \mathbf{BA}$, then \mathbf{AB} is idempotent.

5. If \mathbf{A} is a square matrix satisfying the equation of $\mathbf{A}^3 - 3\mathbf{A} + \mathbf{I} = \mathbf{0}$. Show that \mathbf{A} is invertible, and then find \mathbf{A}^{-1} .

6. Prove if \mathbf{A} is an invertible matrix, then its inverse is unique.

7. Prove that if \mathbf{A} is an $n \times n$ matrix, then $\mathbf{A} - \mathbf{A}^T$ is skew-symmetric.