

Name: _____ Date: _____

Show your work very clearly, neatly, and box your final answer.

One Side Only

1. Consider $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$, $T(x, y, z) = (x, 2y, x + 2y, z - 3y)$.

a. Determine whether the function is a linear transformation.

b. Find the image of $(1, 2, 3)$.

c. Find the preimage of $(1, 1, -2, 2)$.

d. Find its standard matrix A such that $T(\mathbf{v}) = A\mathbf{v}$.

3) The linear transformation T is represented by $T(\mathbf{v}) = A\mathbf{v}$. Find a basis for the kernel of T and the range of T (Range of T is given by the column space of A).

$$A = \begin{bmatrix} -1 & 3 & 2 & 1 & 4 \\ 2 & 3 & 5 & 0 & 0 \\ 2 & 1 & 2 & 1 & 0 \end{bmatrix}.$$

4) The linear transformation T is represented by $T(v) = Av$. Find $\ker(T)$,

range(T), nullity(T), and rank(T). $A = \begin{bmatrix} \frac{4}{9} & \frac{-4}{9} & \frac{2}{9} \\ \frac{-4}{9} & \frac{4}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{-2}{9} & \frac{1}{9} \end{bmatrix}$.