

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Show your work very clearly, neatly, and box your final answer.**

**One Side Only**

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1. Determine if  $T(x, y) = (y, x)$  is a linear transformation. Identify the domain and codomain.

2. Determine if  $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a^2 + b^2$  is a linear transformation. Identify the domain and codomain.

3. Consider  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(x, y, z) = (x - y, y - z, z - x)$ .

a. Determine whether the function is a linear transformation.

b. Find the image of  $(1, 2, 3)$ .

c. Find the pre-image of  $(1, 1, -2)$ .

4. Prove that  $T : V \rightarrow W$  is a linear transformation **if and only if**  
 $T(au + bv) = aT(u) + bT(v)$  for all vectors  $u$  and  $v$  and all scalars  $a$  and  $b$ .