Math 270 HW#20

Name:_______________________________________ Date:_______________

Show your work very clearly, neatly, and box your final answer.

One Side Only

An **inner product** on a vector space V is a function that associates a real number $< u,v >$ with each pair of vectors u and v in V in such a way that the following axioms are satisfied for all vectors $u, v,$ and w in V and all scalars c.

$$1. < u,v > \; = \; < v,u >$$

$$2. < u+v,w > \; = \; < u,w > + < v,w >$$

$$3. < cu,v > \; = \; c < v,u >$$

$$4. < u,u > \; \geq 0$$

$$5. < u,u > \; = 0 \text{ if and only if } u = 0$$

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1. If $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are vectors in R^2, then $< u,v > \; = 3u_1v_1 + u_2v_2$ defines an inner product space. Verify all five axioms stated above.

2. Given the inner product defined by if

$p = a_0 + a_1 x + a_2 x^2 \;\&\; q = b_0 + b_1 x + b_2 x^2$ then

$< p, q > = a_0 b_0 + a_1 b_1 + a_2 b_2$.

Compute the following where $p = -1 + 2x + x^2 \;\&\; q = 2 - 4x^2$

a. $< p, q >$

b. $< q, p >$

c. $< p, p >$

d. $< q, q >$

e. verify $< cp, q > \; = c < p, q >$

If $u = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$ and $v = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$ for any 2×2 matrices, then the following formula defines an inner product space on $M_{2,2}$:

$< u, v > \ = \ u_1 v_1 + u_2 v_2 + u_3 v_3 + u_4 v_4.$

3. Use the above definition of the inner product to find the following for the matrices $u = \begin{bmatrix} 2 & -1 \\ 3 & 7 \end{bmatrix}$ and $v = \begin{bmatrix} 0 & 4 \\ 2 & 2 \end{bmatrix}$.

a. $< u, v >$

b. $< u, u >$

c. $\|u\|$ where $\|u\|^2 = < u, u >$

d. $\|v\|$ where $\|v\|^2 = < v, v >$

f. Find the angle θ by using $\cos\theta = \frac{<u,v>}{\|u\|\|v\|}$.

g. Verify the Cauchy-Schwarz inequality that is $|<u,v>| \le \|u\| \ \|v\|$.

4. Let P_2 have the inner product defined as $<p,q>= a_0b_0 + a_1b_1 + a_2b_2$ if $p = a_0 + a_1x + a_2x^2$ and $q = b_0 + b_1x + b_2x^2$, Determine if $p = 1 - x + x^2$ and $q = 2x + x^2$ are orthogonal or not.

5. Find $d(A,B)$ by using the formula $d(A,B)=\|A-B\|$ for $A=\begin{bmatrix} 1 & 5 \\ 8 & 3 \end{bmatrix}$ and

$B=\begin{bmatrix} -5 & 0 \\ 7 & -3 \end{bmatrix}$ given $<u,v>=u_1v_1+u_2v_2+u_3v_3+u_4v_4$ for

$u=\begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$ and $v=\begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$.

6. Let u and v be vectors in an inner product space V. Prove if

$\|u+v\|^2=\|u\|^2+\|v\|^2$, then u and v are orthogonal vectors.