Math 270 HW#18

Name:____ Date:_______________

Show your work very clearly, neatly, and box your final answer.

One Side Only

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1. Use the inner product defined in R^3 as $< u,v >= u_1v_1 + 2u_2v_2 + 3u_3v_3$ for the vectors $u = (1,-1,0)$ and $v = (6,0,8)$ to find

a. $< u,v >$

b. $< v,v >$

c. $\text{Proj}_v u$

d. $\text{Proj}_u v$

2. Determine if $p(x) = x$ and $q(x) = x^2$ are orthogonal by using the inner product in given by $< p,q >= \int_{-1}^{1} p(x)q(x)dx$.

3. Use $f(x) = x^2$ and $g(x) = e^x$ in $C[-1,1]$, the set of all continuous functions on the interval $[-1,1]$ to find the following using the inner product $< f, g >= \int_{-1}^{1} f(x)g(x)dx$.

a. $< f, g >$

b. $< f, f >$

c. $< g, g >$

4. Prove that $\|u+v\|^2 + \|u-v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$ for any vectors $u \,\&\, v$ in an inner product space V.

5. Let R^2 have the Euclidean inner product (dot product). Apply the Cauchy-Schwarz inequality to the vectors $u=(a,b)$ and $v=(\cos\theta,\sin\theta)$ to show $\left|a\cos\theta+b\sin\theta\right|^2 \le a^2+b^2$.

6. Use the Euclidian inner product (Dot Product) to show the following set is an **orthonormal** set that is all pairs of distinct vectors in the set are **orthogonal** and each vector has **norm 1**.

$$S=\left\{\left(\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6}\right),\left(\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},-\frac{\sqrt{3}}{3}\right)\right\}.$$

7. Are the following vectors linearly dependent or independent?

$$S = \{(2,3,-1),(-5,8,3),(6,-5,1)\}$$

If these vectors are dependent, then find a basis of subspace that is spanned by these vectors.

8. If R^2 has the inner product $<u,v>=3u_1v_1+2u_2v_2$, show that the set below is orthonormal set. $S=\left\{\left(\frac{1}{\sqrt{5}},-\frac{1}{\sqrt{5}}\right),\left(\frac{2}{\sqrt{30}},\frac{3}{\sqrt{30}}\right)\right\}$.

Let u and v be vectors in an inner product space V, then the **orthogonal projection** of u onto v, $v \neq 0$ is given by $\mathrm{proj}_v u = \frac{<u,v>}{<v,v>} v$.

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9. Find the orthogonal projection of f onto g using the inner product in $C[0,1]$ given by $<f,g> = \int_0^1 f(x)g(x)dx$ given $f(x) = x$ and $g(x) = e^{-x}$.