Math 270 HW#17

Name:____ Date:________________

Show your work very clearly, neatly, and box your final answer.

One Side Only

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1. Use the law of cosines and triangle with three sides $\|u\|$, $\|v\|$, and $\|u-v\|$ to prove that $\cos\theta = \frac{u \bullet v}{\|u\|\|v\|}$ where θ is the angle between vectors u and v.

2. Prove the Cauchy-Schwarz Inequality that if u and v are vectors in R^n, then $|u \bullet v| \le \|u\|\|v\|$.

3. Prove the Triangle Inequality that u and v are vectors in R^n, then $\|u+v\| \le \|u\| + \|v\|$

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4. Find the rank and a basis for the row space and the column space of the matrix

$$\begin{bmatrix} 1 & 2 \\ -4 & 3 \\ 6 & 1 \end{bmatrix}.$$

5. Given: $u=(1,2,-1)$ and $v=(0,2,3)$

a. Verify the Cauchy-Schwarz inequality, that is $|u \bullet v| \le \|u\|\|v\|$.

b. Verify the Triangular inequality, that is $\|u+v\| \le \|u\|+\|v\|$.

If $u = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$ and $v = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$ for any 2×2 matrices, with defined an inner product space on $M_{2,2}$: $< u, v > \ = u_1 v_1 + u_2 v_2 + u_3 v_3 + u_4 v_4$.

3. Use the above definition of the inner product to find the following for the matrices $u = \begin{bmatrix} 2 & -1 \\ 3 & 7 \end{bmatrix}$ and $v = \begin{bmatrix} 0 & 4 \\ 2 & 2 \end{bmatrix}$.

a. $< u, v >$

b. $< u, u >$

c. $\|u\|$ where $\|u\|^2 = < u, u >$

d. $\|v\|$ where $\|v\|^2 = < v, v >$

f. Find the angle θ by using $\cos\theta = \dfrac{<u,v>}{\|u\|\|v\|}$.

g. verify the Cauchy-Schwarz inequality that is $|<u,v>| \leq \|u\| \ \|v\|$.