

Name: _____ Date: _____

Show your work very clearly, neatly, and box your final answer.

One Side Only

1. Prove that if a column of square matrix A of order n is multiplied by a nonzero scalar c then $\det(A)$ will be multiplied by the same scalar.

2. Prove If \mathbf{A} is an square skew-symmetric matrix of order n , and n is odd, then $|\mathbf{A}| = 0$.

3. Prove that the determinant of an orthogonal matrix is ± 1 .

4. Prove if matrix A is square matrix of order n , then $|A| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \dots \cdot \lambda_n$.

Determine which sets below are vector spaces under given operations. For those that are not, list all the axioms that fail to hold.

5. The set $\{(2x, x) : x \in \mathbb{R}\}$ with standard operations of addition and scalar multiplication.

6. The set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ a & b \end{bmatrix}$ with standard matrix operations with addition and scalar multiplication.

7. Determine if $W = \{(a, a + b, b) \mid a \in R, b \in R\}$ is the subspace of $V = R^3$.

8. Show that the set W of all 3×3 matrices having zeros on the main diagonal is a subspace of vector space $M_{3,3}$ of all 3×3 matrices with matrix addition and scalar multiplication.

9. Determine if $W = \{(x, y, z) \mid x \geq 0\}$ is the subspace of $V = \mathbf{R}^3$.