Estimating Parameters

and

Proportion Confidence Interval
What is a Parameter?

Parameter is any numerical measurement related to a population.

What are some common Parameters?

Here are some common parameters:

- Population Proportion $p$
- Population Mean $\mu$
- Population Standard Deviation $\sigma$
What do we need to start the **Estimation** process?

We must have a randomly selected sample from the population that has the correct point-estimate.

What is a **Point-Estimate**?

In statistic, the **Point-Estimate** is an **Estimator** of some **Parameter** of the population.

**Point-Estimate** is calculated from the sample data and it is served as a the **Best-Guess** for our estimation of the parameter.
What is a **Confidence Interval**?

In statistics, a **Confidence Interval** is a range of values computed from the statistics of the observed data, that might contain the true value of a population parameter.

Every **Confidence Interval** comes with a **Confidence Level**.

What is a **Confidence Level**?

**Confidence Level** represents the probability that the true parameter lies within the confidence interval.

**Confidence Level** is usually expressed as a percentage.
What are some common **Confidence Levels**?

Here are some common confidence levels:

- 90%
- 95%
- 99%

Important information about **Confidence Levels**:

- When confidence level is not given, use 95%.
- For significance level $\alpha$, where $0 < \alpha < 1$ the confidence level is $(1 - \alpha) \cdot 100\%$. 
<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>( \alpha = 0.1 )</td>
</tr>
<tr>
<td>95%</td>
<td>( \alpha = 0.5 )</td>
</tr>
<tr>
<td>99%</td>
<td>( \alpha = 0.01 )</td>
</tr>
<tr>
<td>( (1 - \alpha) \cdot 100% )</td>
<td>( \alpha, \ 0 &lt; \alpha &lt; 1 )</td>
</tr>
</tbody>
</table>
Confidence Level vs. Significance Level

Display:

\[ \alpha/2 \]

\( (1 - \alpha) \cdot 100\% \)

Confidence Level

\( \alpha/2 \)

C.V.
Confidence Interval for Population Proportion:

- Final Answer: $\cdots < P < \cdots$
- General Format: $\hat{p} - E < P < \hat{p} + E$
- Margin of Error: $E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$
- Sample Proportion: $\hat{p}$ where $\hat{p} = \frac{x}{n}$ and $\hat{q} = 1 - \hat{p}$
- Sample Results: Sample Size $n$ with $x$ favorable responses
- Critical Value: $Z_{\alpha/2}$ for $(1 - \alpha) \cdot 100\%$ confidence level
Example:

In a survey of 850 students, 32% of them were in favor of taking online classes.

- How many students in this survey were in favor of taking online classes?
- Find the critical value for constructing the 90% confidence interval for the proportion of all students that are in favor of taking online classes.
- Find the margin of error when constructing a 90% confidence interval for the proportion of all students that are in favor of taking online classes.
- Find the 90% confidence interval for the proportion of all students that are in favor of taking online classes.
Solution:

Since \( n = 850 \), and \( \hat{p} = 0.32 \),

- How many students in this survey were in favor of taking online classes?
  \[ x = n \cdot \hat{p} = 850 \cdot 0.32 = 272 \]

- Find the critical value for constructing the 90% confidence interval for the proportion of all students that are in favor of taking online classes.

\[ Z_{0.05} = \text{invNorm}(0.95, 0, 1) = 1.645 \]
Solution Continued:

- Find the margin of error when constructing a 90% confidence interval for the proportion of all students that are in favor of taking online classes.

\[ \hat{q} = 1 - \hat{p} = 1 - 0.32 = 0.68 \]

\[ E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = 1.645 \cdot \sqrt{\frac{0.32 \cdot 0.68}{850}} \approx 0.026 \]

- Find the 90% confidence interval for the proportion of all students that are in favor of taking online classes.

\[ \hat{p} - E < P < \hat{p} + E \]

\[ 0.32 - 0.026 < P < 0.32 + 0.026 \]

\[ 0.294 < P < 0.346 \]
Example:

In a survey of 720 students, 495 of them were driving to school alone.

- Find the sample proportion using this survey of students that drive to school alone.
- Find the critical value for constructing a confidence interval for the proportion of all students that drive to school alone.
- Find the margin of error when constructing the confidence interval for the proportion of all students that drive to school alone.
- Find the confidence interval for the proportion of all students that drive to school alone.
Solution:

Since \( n = 720 \), and \( x = 575 \),

1. Find the sample proportion using this survey of students that drive to school alone.
   \[
   \hat{p} = \frac{x}{n} = \frac{495}{720} = 0.6875 \approx 0.688
   \]

2. Find the critical value for constructing a confidence interval for the proportion of all students that drive to school alone.

   Since the confidence level is not given, we use 95%.

   \[
   Z_{0.025} = \text{invNorm}(0.975, 0, 1) = 1.960
   \]
Solution Continued:

- Find the margin of error when constructing the confidence interval for the proportion of all students that drive to school alone.
  \[ \hat{q} = 1 - \hat{p} = 1 - 0.688 = 0.312 \]
  \[ E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = 1.960 \cdot \sqrt{\frac{0.688 \cdot 0.312}{720}} \approx 0.034 \]

- Find the confidence interval for the proportion of all students that drive to school alone.
  \[ \hat{p} - E < P < \hat{p} + E \]
  \[ 0.688 - 0.034 < P < 0.688 + 0.034 \]
  \[ 0.654 < P < 0.722 \]
Finding \( \hat{p} \) & \( E \) from Confidence Interval:

Given the confidence interval \( \text{Lower} < p < \text{Upper} \), then

\[
\hat{p} = \frac{\text{Upper Value} + \text{Lower Value}}{2}
\]

\[
E = \frac{\text{Upper Value} - \text{Lower Value}}{2}
\]
Here are the steps on TI when constructing confidence interval for population proportion:

- STAT
- TESTS
- 1–PropZInt

Pay close attention to the following:

- \( x = n \cdot \hat{p} \), when decimal, always round up.
- When confidence level is not given, use 95%.
- Always round your final answer to three decimal places, and use mathematical notation to display your final answer.
**Example:**

In a survey conducted by the college, 9.4% of 175 randomly selected students were left-handed.

- How many students in this survey were left-handed?
- Find the 99% confidence interval for the proportion of all students that are left-handed.
- Find the margin of error.

**Solution:**

Since \( n = 175 \), and \( \hat{p} = 9.4\% = 0.094 \),

- How many students in this survey were left-handed?

\[ x = n \cdot \hat{p} = 175 \cdot 0.094 = 16.45 \]

Since we have a decimal answer, we round up, therefore \( x = 17 \)
Solution Continued:

- Find the 99% confidence interval for the proportion of all students that are left-handed.

Following the TI commands `STAT > TESTS > 1-PropZInt` with \( x = 17 \), \( n = 175 \), and **C-Level:** 0.99 we get

\[
0.039 < P < 0.155
\]

- Find the margin of error.

\[
E = \frac{\text{Upper Value} - \text{Lower Value}}{2} = \frac{0.155 - 0.039}{2} = 0.058
\]