What is a poisson probability distribution?

It is a discrete probability distribution of random variable \( x \) such that

1. The experiment consists of counting the number of times, \( x \), an event occurs in a given interval.
2. The interval can be an interval of time, area, or volume.
3. The the probability of an event occurring is the same for each interval.
4. The number of occurrences in one interval is independent of the number of occurrences in other intervals.

Then the **probability that the first success will occur on trial number** \( x \) **is**

\[
P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}
\]

Where \( e \approx 2.718 \) and \( \mu \) is the mean number of occurrences per interval and \( \sigma^2 = \mu \).

**Poisson Probability Distribution & TI:**

<table>
<thead>
<tr>
<th>When you have</th>
<th>Use TI command</th>
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</thead>
<tbody>
<tr>
<td>( P(x = a) )</td>
<td>\text{poissonpdf}(\mu, a)</td>
</tr>
<tr>
<td>( P(x \leq a) )</td>
<td>\text{poissoncdf}(\mu, a)</td>
</tr>
<tr>
<td>( P(x \geq a) )</td>
<td>( 1 - \text{poissoncdf}(\mu, a - 1) )</td>
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</table>

**How to find the TI Command:**

2\text{nd} \ VARS, scroll down to \text{poissonpdf} or \text{poissoncdf}, followed by pressing \text{ENTER} button twice.
Example:
The mean number of business failures per day in California in recent year was about 100. Find the probability that

1. exactly 95 businesses will fail in any given day.
2. fewer than 95 businesses will fail in any given day.
3. more than 110 businesses will fail in any given day.
4. between 90 and 110, inclusive, businesses will fail in any given day.

Solution:

1. exactly 95 businesses will fail in any given day. \( \Rightarrow P(x = 95) \)

\[
P(x = 95) = \frac{100^{95} \cdot e^{-100}}{95!}
= \text{poissonpdf}(100, 95)
\approx 0.0360
\]

2. fewer than 95 businesses will fail in any given day. \( \Rightarrow P(x < 95) \)

\[
P(x < 95) = P(x \leq 94)
= \text{poissoncdf}(100, 94)
\approx 0.2952
\]

3. more than 110 businesses will fail in any given day. \( \Rightarrow P(x > 110) \)

\[
P(x > 110) = P(x \geq 111)
= 1 - P(x \leq 110)
= 1 - \text{poissoncdf}(100, 110)
\approx 0.1471
\]

4. between 90 and 110, inclusive, businesses will fail in any given day. \( \Rightarrow P(90 \leq x \leq 110) \)

\[
P(90 \leq x \leq 110) = P(x \leq 110) - P(x \leq 89)
= \text{poissoncdf}(100, 110) - \text{poissoncdf}(100, 89)
\approx 0.7065
\]