Poisson Probability Distribution
What is a **Poisson Probability Distribution**?

It is a probability distribution for a discrete random variable $x$, the number of successes in a fixed interval, with probability $P(x)$ such that the following conditions are met.

- The probability of two or more successes in any sufficiently smaller subinterval is 0.
- The probability of success for any two identical non overlapping interval is the same.
- The number of successes in any interval is independent from from other non overlapping interval with the same length.
- $0 \leq P(x) \leq 1$ and $\sum P(x) = 1$. 
How to find the probability of a Poisson Probability Distribution:

The probability of $x$ successes in an interval of fixed length with mean $\mu$ is

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \text{ for } x = 0, 1, 2, 3, \ldots$$

with $e \approx 2.7183$ and $\sigma^2 = \mu$.

It is common to use the Greek letter lambda $\lambda$ to represent the mean number of occurrences of the event in the given interval.
Example:

Consider a Poisson Probability Distribution for a discrete random variable $x$ with mean $\mu = 10$ on a fixed interval.

- Find $P(x = 5)$.
- Find $P(x < 5)$.
- Find $P(x \geq 5)$.
- Find its variance $\sigma^2$.
- Find its standard deviation $\sigma$. 


Solution:

- Find $P(x = 5) \Rightarrow P(x = 5) = \frac{10^5}{5!}e^{-10} = 0.038$.

- Find $P(x < 5) \Rightarrow P(x < 5) = P(x \leq 4)$.
  \[ \Rightarrow P(x \leq 4) = P(x = 4) + P(x = 3) + \cdots + P(x = 0). \]
  \[ \Rightarrow P(x < 5) = P(x \leq 4) = 0.029 \]

- Find $P(x \geq 5) \Rightarrow P(x \geq 5) = 1 - P(x \leq 4)$.
  \[ \Rightarrow P(x \leq 4) = 0.029. \]
  \[ \Rightarrow P(x \geq 5) = 1 - 0.029 = 0.971 \]

- Find its variance $\sigma^2 \Rightarrow \sigma^2 = \mu = 10$.

- Find its standard deviation $\sigma \Rightarrow \sigma = \sqrt{\sigma^2} = \sqrt{10} \approx 3.162$. 
### Poisson Probability Distributions & TI

When you have | Use TI command
---|---
\( P(x = a) \) | \( \text{poissonpdf}(\mu, a) \)
\( P(x \leq a) \) | \( \text{poissoncdf}(\mu, a) \)
\( P(x \geq a) \) | \( 1 - \text{poissoncdf}(\mu, a - 1) \)

You can find TI commands **poissonpdf** and **poissoncdf** by pressing \( 2\text{ND} \), \( \text{VARS} \), then \( \downarrow \) to locate them.
Example:

The phone calls to the college IT help desk occur at the rate of 1.5 per five minutes between 11:00am to 12:00 noon on Mondays. Compute the probability the number of these calls between 11:30am and 11:45am is

- exactly five.
- fewer than eight.
- at least four.

Solution:

This problem fits all criteria of a Poisson Probability Distribution with \( \mu = 1.5 \cdot 3 = 4.5 \) since the rate is given per five minutes, and our interval is 15 minutes.
Solution Continued:

Let $x$ be the number of calls received by IT help desk during our desired interval.

Now we need to find

- exactly five. \( \Rightarrow P(x = 5) = \text{poissonpdf}(4.5, 5) = 0.171. \)
- fewer than eight.
  \( \Rightarrow P(x < 8) = P(x \leq 7) = \text{poissoncdf}(4.5, 7) = 0.913. \)
- at least four.
  \( \Rightarrow P(x \geq 4) = 1 - P(x \leq 3) \)
  \( \Rightarrow 1 - \text{poissoncdf}(4.5, 3) = 0.658. \)