Venn Diagram & DeMorgan’s Law
What is a Venn Diagram?

Venn Diagram is frequently used by statisticians to display relationships between events in a sample space.

How do we construct a Venn Diagram?

When working with probabilities, the following criteria for constructing Venn Diagram should be followed,

- the sample space is represented by a rectangle.
- events within the sample space are often displayed by circles, clearly labeled by corresponding probabilities.
- the sum of all probabilities within the rectangle is equal to 1.
Example:

Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $A$.

Solution:
Example:

Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $\overline{B}$.

Solution:
Example:
Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $A$ or $B$.

Solution:
Example:
Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $A$ and $B$.

Solution:

![Venn Diagram with Events A and B shaded]
Example:

Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $A$ only.

Solution:
Example:

Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $A$ only or $B$ only.

Solution:
Example:

Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $\overline{A}$ or $\overline{B}$.

Solution:
Now if we superimpose these two Venn diagrams, the only region that is not shaded is the overlap of these two events. When working with OR, we take all the regions that are shaded at least once.
Example:

Construct a Venn Diagram with two events $A$ and $B$, then clearly shade the event $\overline{A}$ and $\overline{B}$.

Solution:
Solution Continued:

Now if we superimpose these two Venn diagrams, the only common shaded region is the outside of both events. When working with **AND**, we take only the commonly shaded region.

\[ \overline{A} \text{ and } \overline{B} \]
What is **DeMorgan’s Law**?

**DeMorgan’s Law** shows a relationship between the union (OR) and the intersection (AND) of the complement of two events.

What are the results of **DeMorgan’s Law**?

In *Mathematical Notation*, it says

- \( P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) \)
- \( P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) \)
Example:

Given: $P(A) = 0.7$, $P(B) = 0.3$, and $P(A \text{ and } B) = 0.2$,
Construct the Venn Diagram using the given information.

Solution:

[Diagram with labels 0.5, 0.2, 0.1, 0.2]
Example:
Use the last example to find \( P(A \text{ only or } B \text{ only}) \).

Solution:
\[
P(A \text{ only or } B \text{ only}) = 0.5 + 0.1 = 0.6
\]
**Example:**

Use the last example to find $P(\overline{A} \text{ or } \overline{B})$.

**Solution:**

Using DeMorgan’s Law, we get

$$P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - 0.2 = 0.8$$
Example:

Use the last example to find $P(\bar{A} \text{ and } \bar{B})$.

Solution:

Using DeMorgan’s Law, we get

$$P(\bar{A} \text{ and } \bar{B}) = P(\bar{A} \text{ or } \bar{B}) = 1 - P(A \text{ or } B) = 1 - 0.8 = 0.2$$