Section 12 Solution

1) Let x be Liz's speed in still water, therefore her speed going downstream is $(x+2)$mph while Paul's speed going upstream is $(x-2)$mph as Liz travels downstream. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Upstream/Paul	$x-2$	t	15
Downstream/Liz	$x+2$	t	27

$$t_{\text{Upstream}}=\frac{15}{x-2}$$
$$t_{\text{Downstream}}=\frac{27}{x+2}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{15}{x-2}=\frac{27}{x+2}$, cross-multiply and solve to get $x=7$. Therefore Paul's speed is $(7-2)=5$mph as she travels downstream.

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2) Let x be the speed of the cruise ship in still water, therefore its speed with the current is $(x+10)$mph while its speed against the current is $(x-10)$mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
With Current	$x+10$	t	275
Against Current	$x-10$	t	175

$$t_{\text{With Current}}=\frac{275}{x+10}$$
$$t_{\text{Against Current}}=\frac{175}{x-10}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{275}{x+10}=\frac{175}{x-10}$, cross-multiply and solve to get $x=45$. Therefore the speed of the cruise ship is $(45+10)=55$mph as it travels with the current.

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3) Let x mph be the speed for Terry, therefore $(x+2)$ mph is the speed for Mark.

Since $t = \frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Terry	x	t	15
Mark	$x+2$	t	21

$t_{\text{Terry}} = \frac{15}{x}$,

$t_{\text{Mark}} = \frac{21}{x+2}$

Now since the amount of time is the same for both, we solve the proportion $\frac{15}{x} = \frac{21}{x+2}$, cross-multiply and solve to get $x = 5$. Therefore the speed for Mark runs at $(5+2) = 7$ mph.

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4) Let x mph be the speed for Doni going to work, therefore $(x-20)$ mph is her speed going to see her friend. Since $t = \frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Work	x	t	100
See friend	$x-20$	t	60

$t_{\text{Work}} = \frac{100}{x}$,

$t_{\text{See friend}} = \frac{60}{x-20}$

Now since the amount of time is the same for both direction, we solve the proportion $\frac{100}{x} = \frac{60}{x-20}$, cross-multiply and solve to get $x = 50$. Therefore Doni is driving to work at 50 mph.

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5) Let x be the speed of the boat in still water, therefore its speed going downstream is $(x+5)$ mph while its speed going upstream is $(x-5)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Upstream	$x-5$	t	135
Downstream	$x+5$	t	165

$$t_{\text{Upstream}}=\frac{135}{x-5}$$
$$t_{\text{Downstream}}=\frac{165}{x+5}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{135}{x-5}=\frac{165}{x+5}$, cross-multiply and solve to get $x=50$. Therefore boat's speed in still water is 50 mph.

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6) Let x be the speed of the steamboat in still water, therefore its speed going downstream is $(x+4)$ mph while its speed going upstream is $(x-4)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Upstream	$x-4$	t	246
Downstream	$x+4$	t	294

$$t_{\text{Upstream}}=\frac{246}{x-4}$$
$$t_{\text{Downstream}}=\frac{294}{x+4}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{246}{x-4}=\frac{294}{x+4}$, cross-multiply and solve to get $x=45$. Therefore steamboat's speed in still water is 45 mph.

Section 12 Solution

7) Let x mph be the speed of the blimp with no wind, therefore its speed with a tailwind is $(x+16)$ mph while its speed with a headwind is $(x-16)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
With tailwind	$x+16$	t	153
With headwind	$x-16$	t	57

$$t_{\text{Tailwind}}=\frac{153}{x+16}$$
$$t_{\text{Headwind}}=\frac{57}{x-16}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{153}{x+16}=\frac{57}{x-16}$, cross-multiply and solve to get $x=35$. Therefore blimp's speed in with no wind is 35 mph.

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8) Let x mph be the speed of the plane with no wind, therefore its speed with the wind is $(x+40)$ mph while its speed against the wind is $(x-40)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
With wind	$x+40$	t	960
Against wind	$x-40$	t	640

$$t_{\text{With wind}}=\frac{960}{x+40}$$
$$t_{\text{Against wind}}=\frac{640}{x-40}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{960}{x+40}=\frac{640}{x-40}$, cross-multiply and solve to get $x=200$. Therefore the plane's speed in with no wind is 200 mph.

Section 12 Solution

9) Let x mph be the speed of the plane with no wind, therefore its speed with a tailwind of 40 mph is $(x+40)$ mph while its speed with a headwind of 20 mph is $(x-20)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Tailwind	$x+40$	t	510
Headwind	$x-20$	t	330

$$t_{\text{Tailwind}}=\frac{510}{x+40}$$
$$t_{\text{Tailwind}}=\frac{330}{x-20}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{510}{x+40}=\frac{330}{x-20}$, cross-multiply and solve to get $x=130$. Therefore the plane's speed in with a headwind is $(130-20)=110$ mph.

10) Let x mph be the speed for the train, therefore $(3x)$ mph is the speed for the plane. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Train	x	t	135
Plane	$3x$	t	855

$$t_{\text{Train}}=\frac{135}{x},$$
$$t_{\text{Plane}}=\frac{855}{3x}=\frac{285}{x}$$

Now since the total time was 6 hrs, we solve the equation $\frac{135}{x}+\frac{285}{x}=6$, multiply by $\text{LCD}=x$ to clear all the fractions and solve to get $x=70$. Therefore the train's speed was 70 mph. **Please refer to the end of this document for more detailed solution.**

Section 12 Solution

11) Let x mph be Harriet's speed in the city, therefore $(x+20)$ mph is her speed on the highway. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
City	x	t	90
Highway	$x+20$	t	130

$t_{\text{City}}=\frac{90}{x}$, $t_{\text{Highway}}=\frac{130}{x+20}$

Now since the total time was 4 hrs, we solve the equation $\frac{90}{x}+\frac{130}{x+20}=4$, multiply by $\text{LCD }=x(x+20)$ to clear all the fractions and solve to get $x=45$. And $x=-10$, but speed cannot be negative. Therefore her speed on the highway was $(45+20)=65$ mph. **<u>Please refer to the end of this document for more detailed solution.</u>**

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12) Let x mph be Frankie 's speed in the paved road, therefore $(x-25)$ mph is his speed on the dirt road. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Paved Road	x	t	135
Dirt Road	$x-25$	t	40

$t_{\text{Paved Rd}}=\frac{135}{x}$, $t_{\text{Dirt Rd}}=\frac{40}{x-25}$

Now since the total time was 5 hrs, we solve the equation $\frac{135}{x}+\frac{40}{x-25}=5$, multiply by $\text{LCD }=x(x-25)$ to clear all the fractions and solve to get $x=45$. And $x=15$, but his speed must be more than 25mph. Therefore his speed on the paved road was 45 mph. **<u>Please refer to the end of this document for more detailed solution.</u>**

Section 12 Solution

13) Let x mph be eagle's speed, therefore pigeon's speed is $(x-5)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Eagle	x	t	100
Pigeon	$x-5$	t	90

$$t_{\text{Eagle}}=\frac{100}{x}$$

$$t_{\text{Pigeon}}=\frac{90}{x-5}$$

Now since the amount of time is the same for both, we solve the proportion $\frac{100}{x}=\frac{90}{x-5}$, cross-multiply and solve to get $x=50$. Therefore an eagle can fly 50 mph while a pigeon can fly 45 mph.

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14) Let x mph be Allen's speed walking, therefore $(x+4)$ mph is his speed jogging. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Walking	x	t	6
Jogging	$x+4$	t	35

$$t_{\text{Walking}}=\frac{6}{x},$$

$$t_{\text{Jogging}}=\frac{35}{x+4}$$

Now since the total time was 7 hrs, we solve the equation $\frac{6}{x}+\frac{35}{x+4}=7$, multiply by $\text{LCD }=x(x+4)$ to clear all the fractions and solve to get $x=3$, and $x=-\frac{8}{7}$, but speed cannot be negative. Therefore Allen was jogging at $(3+4)=7$ mph. **<u>Please refer to the end of this document for more detailed solution.</u>**

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15) Let x mph be the speed of the balloon with no wind, therefore its speed with the wind is $(x+20)$ mph while its speed against the wind is $(x-20)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
With wind	$x+20$	t	200
Against wind	$x-20$	t	40

$$t_{\text{With wind}}=\frac{200}{x+20}$$
$$t_{\text{Against wind}}=\frac{40}{x-20}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{200}{x+20}=\frac{40}{x-20}$, cross-multiply and solve to get $x=30$. Therefore the balloon's speed in against the wind is 10 mph.

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16) Let x be the speed of the raft in still water, therefore its speed going downstream is $(x+7)$ mph while its speed going upstream is $(x-7)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Upstream	$x-7$	t	15
Downstream	$x+7$	t	85

$$t_{\text{Upstream}}=\frac{15}{x-7}$$
$$t_{\text{Downstream}}=\frac{85}{x+7}$$

Now since the amount of time is the same for both direction we solve the proportion $\frac{15}{x-7}=\frac{85}{x+7}$, cross-multiply and solve to get $x=10$. Therefore the speed of the raft going upstream was $(10-7)=3$ mph.

Section 12 Solution

17) Let x mph be Trevor's speed when riding his bicycle , therefore $(x+40)$ mph is his speed riding a motorcycle. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Motorcycle	$x+40$	t	165
Bicycle	x	t	45

$$t_{\text{Motorcycle}}=\frac{165}{x+40},$$
$$t_{\text{Bicycle}}=\frac{45}{x}$$

Now since it took the same amount of time, then we solve the equation $\frac{165}{x+40}=\frac{45}{x}$, we can cross multiply and solve to get $x=15$. Therefore Trevor's speed riding his motorcycle is $(15+40)=55$ mph.

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18) Let x mph be freight train's speed, therefore $(x+35)$ mph is the speed for the passenger train. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Passenger	$x+35$	t	280
Freight	x	t	140

$$t_{\text{Passenger}}=\frac{280}{x+35},$$
$$t_{\text{Freight}}=\frac{140}{x}$$

Now since it took the same amount of time, then we solve the equation $\frac{280}{x+35}=\frac{140}{x}$, we can cross multiply and solve to get $x=35$. Therefore the passenger train travels at $(35+35)=70$ mph.

Section 12 Solution

19) Let x mph be the speed for hiking, therefore $(x+3)$ mph is the speed for walking. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
Walking	$x+3$	t	7
Hiking	x	t	12

$t_{\text{Walking}}=\frac{7}{x+3}$,

$t_{\text{Jogging}}=\frac{12}{x}$

Now since the total time was 4 hrs, we solve the equation $\frac{7}{x+3}+\frac{12}{x}=4$, multiply by $\text{LCD}=x(x+3)$ to clear all the fractions and solve to get $x=4$, and $x=-\frac{9}{4}$, but speed cannot be negative. Therefore tourists were hiking at 4 mph.

<u>Please refer to the end of this document for more detailed solution.</u>

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20) Let x mph be Karl's speed with no current, therefore his speed with the current is $(x+5)$ mph while his speed against the current is $(x-5)$ mph. Since $t=\frac{d}{r}$, we are now ready to set up our chart.

Categories	Rate	Time	Diatance
With Current	$x+5$	t	33
Against Current	$x-5$	t	3

$t_{\text{With Current}}=\frac{33}{x+5}$

$t_{\text{Against Current}}=\frac{3}{x-5}$

Now since the total time was 6 hrs, we solve the equation $\frac{33}{x+5}+\frac{3}{x-5}=6$, multiply by $\text{LCD}=(x+5)(x-5)$ to clear all the fractions and solve to get $x=6$, and $x=0$, but speed of zero does not work. So Karl was swimming at $(6+5)=11$ mph with the current. **<u>Please refer to the end of this document for more detailed solution.</u>**

Detailed solutions for problems 10, 11, 12, 14, 19, and 20:

10)

$\frac{135}{x}+\frac{285}{x}=6,$ Multiply by LCD $= x$ to clear fractions

$135+285=6x,$ Simplify

$420=6x,$ Solve for x by dividing both sides by 6

$\frac{420}{6}=x,$

$70=x$

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11)

$\frac{90}{x}+\frac{130}{x+20}=4,$ Multiply by LCD $= x(x+20)$ to clear fractions

$90(x+20)+130x=4x(x+20),$ Simplify

$90x+1800+130x=4x^2+80x,$ Collect like terms,

$220x+1800=4x^2+80x,$ Since this a quadratic equation we make one side zero,

$0=4x^2+80x-220x-1800,$ Simplify again

$4x^2-140x-1800=0,$ Divide by 4 since all the numbers are divisible by 4

$x^2-35x-450=0,$ Factor the left hand side

$(x-45)(x+10)=0,$ Use the zero-product principle and solve each factor

$x-45=0$ or $x+10=0$

$x=45$ or $x=-10$

12)

$\frac{135}{x}+\frac{40}{x-25}=5,$ Multiply by LCD $= x(x-25)$ to clear fractions

$135(x-25)+40x=5x(x-25),$ Simplify

$135x-3375+40x=5x^2-125x,$ Collect like terms,

$175x-3375=5x^2-125x,$ Since this a quadratic equation we make one side zero,

$0=5x^2-125x-175x+3375,$ Simplify again

$5x^2-300x+3375=0,$ Divide by 5 to simplidy again

$x^2-60x+675=0,$ Factor the left hand side

$(x-15)(x-45)=0,$ Use the zero-product principle and solve each factor

$x-15=0$ or $x-45=0$

$x=15$ or $x=45$

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14)

$\frac{6}{x}+\frac{35}{x+4}=7,$ Multiply by LCD $= x(x+4)$ to clear fractions

$6(x+4)+35x=7x(x+4),$ Simplify

$6x+24+35x=7x^2+28x,$ Collect like terms,

$41x+24=7x^2+28x,$ Since this a quadratic equation we make one side zero,

$0=7x^2+28x-41x-24,$ Simplify again

$7x^2-13x-24=0,$ Factor the left hand side

$(7x+8)(x-3)=0,$ Use the zero-product principle and solve each factor

$7x+8=0$ or $x-3=0$

$x=\frac{-8}{7}$ or $x=3$

19)

$$\frac{7}{x+3}+\frac{12}{x}=4,\ \text{Multiply by LCD} = x(x+3) \text{ to clear fractions}$$

$$7x+12(x+3)=4x(x+3),\ \text{Simplify}$$

$$7x+12x+36=4x^2+12x, \text{Collect like terms,}$$

$$19x+36=4x^2+12x, \text{Since this a quadratic equation we make one side zero,}$$

$$0=4x^2+12x-19x-36, \text{Simplify again}$$

$$4x^2-7x-36=0, \text{Factor the left hand side}$$

$$(4x+9)(x-4)=0, \text{Use the zero-product principle and solve each factor}$$

$$4x+9=0 \text{ or } x-4=0$$

$$x=\frac{-9}{4} \text{ or } x=4$$

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20)

$$\frac{33}{x+5}+\frac{3}{x-5}=6,\ \text{Multiply by LCD} = (x+5)(x-5) \text{ to clear fractions}$$

$$33(x-5)+3(x+5)=6(x+5)(x-5),\ \text{Simplify}$$

$$33x-165+3x+15=6x^2-150, \text{Collect like terms,}$$

$$36x-150=6x^2-150, \text{Since this a quadratic equation we make one side zero,}$$

$$0=6x^2-150-36x+150, \text{Simplify again}$$

$$6x^2-36x=0, \text{Divide by 6 to simplidy again}$$

$$x^2-6x=0, \text{Factor the left hand side}$$

$$x(x-6)=0, \text{Use the zero-product principle and solve each factor}$$

$$x=0 \text{ or } x-6=0$$

$$x=0 \text{ or } x=6$$