Section 7

Motion – Linear Equations

For some students, these are the problems that give them the worst headaches. You have trains, planes, buses, joggers, or anything that moves, going in the same direction, going in the opposite direction – and then you're asked question like:

- How long will it take before they meet?
- How long will it be before they're 200 miles apart?
- How fast were they going?

When you finish this section, you will learn how to pick out the important bits of information and what to do with it so that you will be able to make sense of these problems and solve them.

Step 1
Read Through The Entire Problem

In these Motion Problems, you have two objects in motion traveling in the same or opposite directions. You need to look for and take note of any information that refers to

- the **time of travel**
- the **speed of the objects**
- the **miles apart the objects are**
- **the direction** the objects are moving (the same or opposite)

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Step 2
Fill In The Chart Using The Distance Formula

Motion problems rely heavily on the distance formula, "**distance (*d*) = rate (*r*) • time (*t*)**".

Drawing a pre-equation chart like the one below will organize your information so that you can set up your equation. The chart always has two rows that represent the two objects in motion.

r	•	***t***	=	***d***

Enter in your chart the information in the problem that refers to **rate** (speed) and **time**.

The **rate in the problem** is given in one of two ways. It is either given as a numerical speed (for example, 35 mph) or you will need to use Direct Translation to name an expression for each rate.

In this section, **the amount of time for both moving objects** will always be the same.

- If the amount of time is given, put that same amount in the Time Column (t) for both rows.
- If the amount of time is not given, use the same variable for both.

Distance is not given in the word problem. You will get the expression to fill in the distance portion of the chart by using the distance formula and multiplying together the rate and time you enter.

HELPFUL HINT

- Do not confuse **Distance** with **Miles Apart**. Although "Miles Apart" represents a distance between two objects and IS given in the word problem, the "distance" in the formula represents how far each of the object has traveled. You will be using the amount of "Miles Apart" in the next step to set up the equation .

Step 3
Set Up the Equation Based on the Information in the Chart

Now that you have created your chart, you have a snapshot of the word problem and the important information you need to set up the equation. It is important to note that not everything from your chart goes into the equation.

To determine the equation, you will need the two distances from the chart and the "miles apart" that you are given in the problem.

For the Motion Problems in this section, there are only **two possible ways to set up an equation**.

- If they are traveling in **opposite directions**, you add the two distances from the chart set equal to the miles apart.
- If they are traveling in the **same direction**, you *subtract the smaller distance from the larger* set equal to the miles apart.

Direction	Equation
Opposite	Distance + distance = miles apart
Same	Distance – distance = miles apart

HELPFUL HINT

- It is important to be aware that opposite does not necessarily mean traveling away from each other. Even though objects are going toward each other, *they are still moving in opposite directions.*

Step 4
Solve the Equation

Using the method taught by your instructor, solve the equation for the variable.

Step 5
Make Sure to Answer the Question Being Asked

In Motion Problems, as in other word problems, you need to make sure exactly what question is being asked. It is possible that the value for the variable x may be your answer. But it may *not* be.

For example, the value for x may be the time, or it may be one of the two rates. If the question asks for "time", you are done. The variable x will be the answer.

If the question asks for "rate", you need to see which rate is being asked for in the problem, and substitute the value of x in that rate in the chart.

EXAMPLES

EXAMPLE 1 Two steamboats leave a city at the same time traveling in the same direction on a straight canal. One travels at 18 miles per hour and the other travels at 25 miles per hour. In how many hours will the boats be 35 miles apart?

SOLUTION

Step 1 *Read The Problem*

- There are two objects in motion, traveling; it is a Motion – Linear Equation Problem.
- The rate (speed) of each object is given. The miles apart is given.
- The time is *not* given; that will be your variable.
- The two objects are traveling in the *same* direction.

Step 2 *Fill In The Chart*

- In your chart, fill in the rates given for each steamboat. One rate is 18 and one is 25.
- In your chart, fill in the time for both steamboats with the variable x.
- In your chart, fill in the distance for each steamboat by multiplying its rate and time.

Object	***r***	•	***t***	=	***d***
Steamboat 1	18		x		$18x$
Steamboat 2	25		x		$25x$

Step 3 *Set Up The Equation*

- The problem states the steamboats are moving in the *same* direction.
- Use the expressions for the two distances that you named in your chart in Step 2.
- Miles Apart is given in the problem. It is 35.
- Set up your equation as distance – distance = miles apart.

$$25x - 18x = 35$$

Step 4 *Solve The Equation*

- The solution to the equation is $\boxed{x = 5}$

Step 5 *Answer The Question Asked*

- You have the solution to the equation. The value of x is 5.
- The question asked is "how many hours?" The question is asking for the time.
- When the question asks for "time", you are done. The value of x is the answer.

Answer: In 5 hours

EXAMPLE 2 Two trains leave the station at the same time. One train is traveling North at 10 mph faster than the other train which is traveling South. After 6 hours, the two trains are 720 miles apart. At what speed did the faster train travel?

SOLUTION

Step 1 *Read The Problem*

- There are two objects in motion, traveling; it is a Motion – Linear Equation Problem.
- The time is given. The miles apart is given.
- The rates are *not* given. You will name the expressions for the rates.
- The two objects are traveling in *opposite* directions.

Step 2 *Fill In The Chart*

- In your chart, fill in the same time given for each train. It is 6 hours.
- The rate of the slower train is the Totally Unknown. Use the variable x.
- Use Direct Translation to name the expression for the rate of faster train. It is $x + 10$.
- In your chart, fill in the distance for each train by multiplying its rate and time.

Object	***r*** • ***t*** = ***d***		
Slower Train	x	6	$6x$
Faster Train	$x + 10$	6	$6(x + 10)$

Step 3 *Set Up The Equation*

- The problem states the trains are moving in *opposite* directions.
- Use the expressions for the two distances that you named in your chart in Step 2.
- Miles Apart is given in the problem. It is 720.
- Set up your equation as distance + distance = miles apart.

$$6x + 6(x + 10) = 720$$

Step 4 *Solve The Equation*

- The solution to the equation is $x = 55$

Step 5 *Answer The Question Asked*

- You have the solution to the equation. But that is NOT the answer to the problem.
- The value of x (55) is for the rate of the slower train.
- The question asked for is "the rate of the faster train".
- Substitute 55 for x into the expression in your chart for the rate of the faster train.

$$\begin{aligned} \text{Faster train} &= x + 10 \\ \text{Faster train} &= 55 + 10 \\ \text{Faster train} &= 65 \end{aligned}$$

Answer: The rate is 65 mph

Motion – Linear Equations: Exercise Set

1. Beth and Jade begin rollerblading at the same time going in opposite directions. Beth is traveling at 10 mph, and Jade is traveling at 6 mph. How long will it be before they are 32 miles apart?

2. Two planes leave Miami International Airport at the same time flying in opposite directions. One plane is traveling at a speed of 550 mph and the other plane is traveling at a speed of 600 mph. In how many hours will they be 5750 miles apart?

3. Two thoroughbred horses begin a marathon at the same time, heading in the same direction. One horse gallops at a speed of 7 mph while the other horse gallops at a speed of 12 mph. How long will it be before the two greyhounds are 10 miles apart?

4. On University Drive in Lauderhill, two buses leave the bus stop at the same time traveling in the same direction. One bus is traveling at 40 mph and the other bus is traveling at 45 mph. How long will it be before they are 30 miles apart?

5. Two trains leave a city at the same time. One travels north at 60 mph and the other travels south at 80 mph. In how many hours will they be 420 miles apart?

6. Sharon and Bentley leave BCC at the same time traveling in cars going in opposite directions. Sharon travels at 40 mph and Bentley travels at 60 mph. In how many hours will they be 350 miles apart?

7. Trent and Evan are 72 miles apart. Both start riding their bicycles at the same time and travel toward each other. If Trent bikes at 13 mph and Evan bikes at 11 mph, in how many hours will they meet?

8. Two cross country skiers are 45 miles apart at opposite ends of a course. The two skiers start at the same time and travel towards each other. One skier is traveling at 14 mph and the second skier is traveling at 16 mph. How long will it take them before they meet?

9. John Elway left Denver on a plane flying at a speed of 350 mph traveling northwest to Seattle. His coach, Mike Shanahan, left Denver at the same time on a plane flying at a speed of 420 mph traveling southeast to Atlanta. How many hours did it take until their flights were 1925 miles apart?

10. Two high school friends leave Fort Lauderdale at the same time to go to college. One is driving North to Gainesville at a speed of 65 mph. Her friend is driving South to the University of Miami at a speed of 55 mph. How long will it be before the two friends are 180 miles apart?

11. Two joggers are 10 miles apart. At the same time, then begin running towards each other. One jogger is running at 4 mph and the other is running at 6 mph. How long after they begin will they meet?

12. An airplane leaves Kennedy Airport in New York headed for Los Angeles International Airport in California. The plane is flying at a speed of 450 mph. Another airplane leaves Los Angeles International Airport headed for Kennedy Airport in New York, flying at a speed of 500 mph. If the two airports are 2375 miles apart, when will the two planes pass each other in flight?

13. The band members of Walton High School in the Bronx are going to a band competition in Florida. They travel on two buses that leave at the same time going in the same direction. One bus is traveling at a speed of 55 mph and the other is driving at a speed of 45 mph. How long will it take for the buses to be 50 miles apart?

14. A freight train and a passenger train both leave a train station at the same time, traveling in the same direction. The speed of the freight train is 45 mph and the speed of the passenger train is 65 mph. How long will it take before the two trains are 70 miles apart?

15. Victor and Ingrid leave their house at the same time riding their bicycles in opposite directions. Victor rides 5 mph faster than Ingrid. After two hours, they are 58 miles apart. At what speed was Victor riding?

16. Frances and Larry live 306 miles apart. They start driving towards each other and meet in three hours. If Frances drives 12 mph faster than Larry, find Larry's speed.

17. Two hot air balloons are 140 miles apart and are traveling towards each other. One hot air balloon is traveling 15 mph faster than the other. After 4 hours of traveling, they meet. What was the speed of each hot air balloon?

18. Vivian and her sister, Vicky, start jogging at the same time, going in opposite directions. Vivian jogs 3 mph faster than Vicky. After jogging for 3 hours, they are 39 miles apart. At what speed were both sisters jogging?

19. Randi and her daughter, Sierra, are white water rafting in Colorado. At the start of the day they are 160 miles apart. They begin traveling toward each other at the same time. Sierra's raft is traveling 8 mph faster than Randi's. If they meet in 5 hours, how fast was Randi's raft traveling?

20. Two cross country skiers begin at the same time, traveling in opposite directions. One skier is traveling at a rate of 6 mph slower than the other skier. In 8 hours, they are 96 miles apart. At what speed was the faster skier traveling?